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Uniqueness of a recently proposed U(1) gauge extension

Ernest Ma

Physics Department, University of California, Riverside, California 92521, USA

Abstract

Consider the addition of a right-handed $SU(2)$ fermion multiplet (with neither color nor hypercharge) to each family of quarks and leptons. The resultant theory admits a new U(1) gauge symmetry only if the additional multiplet is a singlet N_R, which corresponds to the well-known case of $U(1)_{B-L}$, or a triplet $(\Sigma^+, \Sigma^0, \Sigma^-)$, which corresponds to the proposal of hep-ph/0112232. This disproves the assertion that the latter is somehow a "trivial" or "expected" discovery.

Consider $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ as a possible extension of the standard model, under which each family of quarks and leptons transforms as follows:

$$
\begin{aligned}
&(u,d)_L \sim (3,2,1/6;n_1), \quad u_R \sim (3,1,2/3;n_2), \quad d_R \sim (3,1,-1/3;n_3), \\
&(\nu,e)_L \sim (1,2,-1/2;n_4), \quad e_R \sim (1,1,-1;n_5).
\end{aligned} \tag{1}
$$

Add to these a right-handed fermion multiplet $(1, 2p{+}1, 0; n_6)$, where $p = 0$ would correspond to a singlet, say N_R, as in the usual extension to include a right-handed neutrino singlet, and $p = 1$ would correspond to a triplet $(\Sigma^+, \Sigma^0, \Sigma^-)_R$, as proposed recently [1].

As shown in Ref. [1], there are 6 conditions to be satisfied for the gauging of $U(1)_X$. Three of them do not involve n_6 and have 2 solutions:

$$
\text{(I)}: \qquad n_3 = 2n_1 - n_2, \quad n_4 = -3n_1, \quad n_5 = -2n_1 - n_2; \tag{2}
$$

$$
\text{(II)}: \qquad n_2 = \frac{1}{4}(7n_1 - 3n_4), \quad n_3 = \frac{1}{4}(n_1 + 3n_4), \quad n_5 = \frac{1}{4}(-9n_1 + 5n_4). \tag{3}
$$

The other 3 involve n_6, and they are given by

$$
\frac{1}{2}(3n_1 + n_4) = \frac{1}{3}p(p+1)(2p+1)n_6, \tag{4}
$$

$$
6n_1 - 3n_2 - 3n_3 + 2n_4 - n_5 = (2p+1)n_6, \tag{5}
$$

$$
6n_1^3 - 3n_2^3 - 3n_3^3 + 2n_4^3 - n_5^3 = (2p+1)n_6^3. \tag{6}
$$

To find solutions to the above 3 equations, consider first $p = 0$, then Eq. (4) forces one to choose solution (I), and all other equations are satisfied with $n_1 = n_2 = n_3$ and $n_4 = n_5 = n_6$, i.e. $U(1)_{B-L}$ has been obtained. Consider now $p \neq 0$, then if solution (I) is again chosen, $n_6 = 0$ is required, which leads to $U(1)_Y$, so there is nothing new.

Now consider $p \neq 0$ and solution (II). From Eqs. (4), (5), and (6), it is easily shown that

$$
\frac{4n_6}{3n_1 + n_4} = \frac{6}{p(p+1)(2p+1)} = \frac{3}{2p+1} = \left(\frac{3}{2p+1}\right)^{\frac{1}{3}}, \tag{7}
$$

which clearly gives the unique solution of $p = 1$, i.e. a triplet. The fact that such a solution even exists (and for an integer value of p) for the above overconstrained set of conditions is certainly not a "trivial" or even "expected" result.

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References

[1] E. Ma, hep-ph/0112232.